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CRT-856

Vehicle Routing with Stochastic Demands and Split Deliveries

by

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Centre de recherche sur les transports — Publication #856

October 1992

Dépôt légal – Bibliothèque nationale du Québec,
Bibliothèque nationale du Canada, 1992

93-01793/

ABSTRACT

This paper describes a heuristic algorithm for vehicle routing problems combining stochastic demands and split deliveries. The algorithm is tested on problems containing up to 130 demand points. For some demand parameters, allowing split deliveries in a stochastic context can yield substantial savings.

Key words : vehicle routing problem, stochastic demands, split deliveries.

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RÉSUMÉ

On décrit dans cet article un algorithme heuristique pour des problèmes de tournées avec demandes à la fois stochastiques et fractionnables. Des tests sont effectués sur des problèmes comportant jusqu'à 130 points de livraison. Pour certains paramètres de demande, on peut réaliser des économies substantielles en permettant des livraisons fractionnées dans un contexte stochastique.

Mots-clés : problèmes de tournées de véhicules, demandes stochastiques, livraisons fractionnées.

1 INTRODUCTION

The *Vehicle Routing Problem* (VRP) arises in a wide variety of distribution planning contexts. It can be described as follows. Consider a set $\{1,...,n\}$ of cities or customers, where the first city is the depot. With every city $i > 1$ is associated a non-negative demand. Let (c_{ij}) be matrix of distances or travel costs between city pairs. There are at most m identical vehicles of capacity D based at the depot. The *capacity constrained VRP* consists of determining a set of m vehicle delivery routes of total minimum cost satisfying the following constraints:

(i) each route starts and ends at the depot;

(ii) every remaining city is visited exactly once by exactly one vehicle;

(iii) the total demand of any vehicle route does not exceed D.

In some contexts, customers have supplies instead of demands and vehicles then make collections.

There exist several other types of VRPs, the most common being the distance constrained VRP (Laporte *et al.*, 1984) and the VRP with time windows. Comprehensive surveys can be found in Bodin *et al.*, (1983), Christofides (1985), Laporte and Nobert (1987), and Laporte (1992). In recent years, some of the research effort has shifted from the classical VRP to more realistic variants of the problem. A number of interesting cases are reported in the book edited by Golden and Assad (1988). We are interested here in the *VRP with stochastic demands* and in the *VRP with split deliveries*.

Stochastic VRPs occur in contexts where some of the problem data are random variables; in our case we assume random demands. Typical examples of this class of problems are the delivery of home heating oil, or of fuel to service-stations, and coin collection from telephone booths. Larson (1988) describes an application related to sludge disposal. In such contexts, vehicle routes may not be followed as planned since total demand of a planned route may turn out to exceed the vehicle capacity; this has direct consequences on problem definition and on solution methodology. For recent studies on stochastic VRPs, see Dror and Trudeau (1986), Laporte *et al.* (1989) and Dror *et al.* (1989). In the VRP with split deliveries, it is no longer assumed that every city is served by exactly one vehicle. Instead, its demand may be shared by several vehicles. Again, oil or fuel delivery is an area where split delivery routing makes sense. Dror and Trudeau (1990) have studied this case in deterministic contexts and have shown that allowing split deliveries may generate substantial savings. An exact algorithm for the VRP with split deliveries is described in Dror, Laporte and Trudeau (1993).

As far as the authors are aware, no study has ever been made on VRPs combining stochastic demands and split deliveries. This a particularly important and realistic combination for the following reason. In a stochastic context, route failures are common. These occur when a vehicle becomes empty in the process of fulfilling a customer's demand. In such a case, several recourse actions are possible, such as returning to the depot for replenishment and resuming deliveries at the point of failure. Such a policy may of course be suboptimal, and it could be preferable for the vehicle involved to move to a different customer and to

let another vehicle serve the customer whose delivery could not be completed. In other words, split deliveries could prove a natural and economical answer to route failures.

The object of this paper is to describe a heuristic algorithm for VRPs with stochastic demands and split deliveries. Extensive computational results confirm the efficiency and the robustness of the proposed approach.

2 ALGORITHM

The algorithm is constructed around the heuristic procedure developed by Dror and Trudeau (1986) for the Stochastic VRP, itself derived from the Clarke and Wright (1964) algorithm for the deterministic VRP. We first describe the Dror and Trudeau algorithm.

STOCH : An algorithm for the stochastic VRP

Let ξ_i be a discrete or continuous non-negative independent random variable denoting the demand of customer i and, for the sake of simplicity, assume $\xi_i \leq D$ for all i. If this not the case, artificial customers can be created by appropriately splitting the demands of customers for which $\xi_i > D$. Each ξ_i has finite mean μ_i and a finite variance σ_i^2 and we assume the variables $(\xi_i - \mu_i)/\sigma_i$ to be identically distributed. The problem consists of determining a set of m ***planned*** vehicle routes. Each route is followed by a vehicle until the sum of demands of all visited customers exceeds D. At this point, ***failure*** is said to occur and the following ***recourse policy*** is applied. All remaining customers on the route whose demand has not yet been satisfied are visited by means of return trips to and from the depot. This policy penalizes heavily the occurrence of failure on a planned route. Under

these rules and assumptions, the probability of failure at any customer along a planned route can easily be computed – see Dror and Trudeau (1986), for example – and the expected cost of any route (including the planned cost and the cost of recourse action) can therefore be derived. The objective is to determine a set of vehicle routes of least total expected costs. In order to solve this problem, Dror and Trudeau (1986) have suggested an adaptation of the Clarke and Wright (1964) savings algorithm. In the original algorithm, the decision whether to combine two routes with customers i and j respectively into a single route depends on the value of the savings $s_{ij} = c_{i1} + c_{1j} - c_{ij}$ and on the feasibility of such an operation. In STOCH, the savings term is replaced by $\max\left\{c'_i + c'_j - c'_{ij}, c'_i + c'_j - c'_{ji}\right\}$, where c'_i is the expected cost of the current route including i, and c'_{ij} is the expected cost of the combined route where i immediately precedes j. ■

SPLIT-STOCH: Modification of the above algorithm to allow for split deliveries

The modified algorithm consists of repeatedly executing the above STOCH procedure (adapted for stochastic problems), by allowing customers to be split. Each execution of this procedure corresponds to one iteration. At the first iteration, the STOCH procedure is executed without modifications, and the optimal value z^* is recorded. At the following iterations, the program identifies the last r cities that were linked together by an arc in the route merging process. Each of these cities i is then split into s cities having demands of means μ_i/s and variances σ_i^2/s. The next iterations are then executed with the following modification: routes consisting of single cities are only merged if these really constitute different cities, and not

"fractions" of the same cities. This process is repeated as long as it produces a decrease in the objective function. If no improvement is noted, it stops. ■

The values of r and s were determined experimentally by performing various sensitivity analyses. Appropriate details on this are provided in Section 3. In addition, we have experimented with the following variant of the algorithm. The vehicle capacity is artificially reduced to tD, where $0 < t \leq 1$. After each pass of STOCH, the cost of route failure is computed using the original capacity D. The use of a reduced capacity helps to some extent prevent solutions containing too many route failures. The choice of t is also discussed in the following section.

3 COMPUTATIONAL RESULTS

The algorithm was tested on 1260 problems generated as follows. Ten sets of coordinates were first generated for each of the three sizes n = 75, 115, and 130. All problems were obtained by randomly generating coordinates in $[0, 80]^2$ according to a uniform distribution. The vehicle capacity was set at 160 in all cases. For each size and for each set of coordinates, fourty-two problems were obtained by generating six sets of mean demands μ_i, according to a uniform distribution in $[160\alpha, 160\beta]$, for each of the seven cases $[\alpha, \beta] = [.1, .3], [.1, .5], [.1, .7], [.1, .9], [.3, .7], [.3, .9]$ and $[.5, .9]$. The value of σ_i was set equal to $(T\mu_i/23)^{1.4}$, where T was randomly generated in $[0, 1]$ according to a uniform distribution. The program was written in FORTRAN, and was run on a Silicon Graphics computer.

To determine the value of s, we first set n equal to 75, r equal to 16 and t equal to 1. The value of s was successively set at 2, 4, 7 and 12 and 10 problems

were generated for each of the seven $[\alpha, \beta]$ intervals. In all cases, the best results were obtained for $s = 4$. To choose the best value of r, parameter s was fixed at 4 and t at 1. Ten problems were then solved for $n = 10$, 115 and 130, all seven intervals $[\alpha, \beta]$, and $r = 10, 16, 20, 25, 35$. Test results indicate that the best r is equal to 20 or 25 if $n = 75$, to 25 or 35 if $n = 115$, and to 35 if $n = 130$. It was therefore decided to set r equal to $[.3n]$. Finally, the best value of t was found to be equal to .99 after having run random problems for $n = 75$, 115 and 130, all seven intervals $[\alpha, \beta]$, $s = 4$, $r = [.3n]$, and $t = .8, .85, .9, .95, .98, .99, 1$.

All subsequent tests were executed with $s = 4$, $r = [.3n]$ and $t = .99$. Computational results are summarized in Table 1. The various column headings can be interpreted as follows:

n : number of cities;

$[\alpha, \beta]$: control parameters for demand generation;

ITR : number of iterations;

TIME : total computation time in seconds;

COST IMPROVEMENT: value of $(z_2 - z_1)/z_1$, where z_1 = expected solution value after the first iteration, and z_2 = final expected solution value;

- AVERAGE : mean value for all 60 problems with a given n and a given $[\alpha, \beta]$;
- MAXIMUM : largest value over 60 problems;
- FREQUENCY : percentage of problems for which a positive improvement was observed

ROUTE IMPROVEMENT : value of $(m_2 - m_1)/m_1$ where m_1 = number of vehicle routes after the first iteration, and m_2 = number of vehicle routes in the final solution. The three columns AVERAGE, MAXIMUM and FREQUENCY are interpreted as above.

The columns ITR, COMPUTATION TIME and AVERAGE represent mean values over 60 problems.

Our results indicate that on the average, the algorithm terminated after a limited number of iterations (less than four on the average). The average gain achieved in routing costs is highly dependent on the interval $[\alpha, \beta]$ used as a basis to generate the mean demands. (e.g., those corresponding to $[\alpha, \beta] = [.5, .9]$ are likely to benefit more from splitting).

Although the algorithm makes no specific attempt at reducing the number of vehicle routes, significant vehicle savings are generated as a by-product. Savings in excess of 15% are frequently observed; these can be as large as 39%. There is therefore a distinct advantage in allowing for split deliveries in a stochastic context. This is consistent with the conclusion reached by Dror and Trudeau (1990) for deterministic problems.

ACKNOWLEDGEMENT

This work was partially supported by the Canadian Natural Sciences and Engineering Research Council (grants OGP0036429 and OGP0039682).

Table 1

Summary of computational results

n	$[\alpha, \beta]$	*ITR.*	*COMPUTATION TIME (sec.)*	*COST IMPROVEMENT*			*ROUTE IMPROVEMENT*		
				AVERAGE	*MAXIMUM*	*FREQUENCY*	*AVERAGE*	*MAXIMUM*	*FREQUENCY*
75	[.1,.3]	1.3	312.0	0.3%	3.1%	26.7%	1.3%	13.3%	18.3%
	[.1,.5]	1.8	649.5	1.3%	5.8%	65.0%	4.6%	13.6%	56.7%
	[.1,.7]	2.5	1258.7	4.6%	9.9%	100.0%	12.0%	22.2%	91.7%
	[.1,.9]	2.8	1243.7	4.1%	8.3%	100.0%	10.6%	17.9%	96.7%
	[.3,.7]	2.9	1462.7	8.2%	15.3%	100.0%	14.7%	26.2%	91.7%
	[.3,.9]	2.7	1535.7	7.3%	12.7%	100.0%	13.8%	24.4%	91.7%
	[.5,.9]	3.7	1800.2	13.7%	20.2%	100.0%	26.4%	39.6%	95.0%
115	[.1,.3]	1.5	1140.9	0.4%	1.6%	53.3%	2.4%	8.7%	43.3%
	[.1,.5]	2.0	1858.5	1.5%	3.9%	86.7%	6.7%	11.8%	85.0%
	[.1,.7]	2.4	2815.9	5.2%	10.8%	100.0%	11.7%	18.8%	93.3%
	[.1,.9]	2.6	3877.1	5.7%	9.9%	100.0%	11.6%	17.5%	96.7%
	[.3,.7]	2.8	3739.5	9.3%	14.4%	100.0%	15.1%	26.2%	90.0%
	[.3,.9]	2.8	3869.2	8.9%	13.6%	100.0%	15.2%	22.2%	96.7%
	[.5,.9]	3.5	5975.0	16.6%	22.1%	100.0%	27.0%	35.7%	95.0%
130	[.1,.3]	1.5	1323.0	0.5%	2.7%	48.3%	2.5%	11.5%	45.0%
	[.1,.5]	2.1	2645.4	1.5%	4.0%	93.3%	7.2%	12.5%	91.7%
	[.1,.7]	2.4	3445.0	5.1%	10.7%	100.0%	11.6%	20.8%	95.0%
	[.1,.9]	2.6	4275.8	5.9%	9.8%	100.0%	12.8%	19.4%	100.0%
	[.3,.7]	2.8	5081.4	9.8%	14.8%	100.0%	15.9%	24.6%	96.7%
	[.3,.9]	2.8	4996.6	9.4%	12.2%	100.0%	15.4%	21.7%	98.3%
	[.5,.9]	3.8	10328.9	17.8%	22.8%	100.0%	27.5%	39.1%	91.7%

REFERENCES

Bodin, L.D., B.L. Golden, A.A. Assad and M.O. Ball (1983), "Routing and Scheduling of Vehicles and Crews. The State of Art." *Computers and Operations Research* 10, 67–211.

Christofides, N. (1985), "Vehicle Routing", in *The Traveling Salesman Problem: A Guided Tour of Combinatorial Optimization*, pp. 431–448, E.L. Lawler, J.K. Lenstra, A.H.G. Rinnooy Kan and D.B. Shmoys (eds.), Wiley, Chichester.

Clarke, G. and J.W. Wright (1964), "Scheduling of Vehicles from a Central Depot to a Number of Delivery Points", *Operations Research* 12, 568–581.

Dror, M., G. Laporte and P. Trudeau (1989), "Vehicle Routing with Stochastic Demands: Properties and Solution Frameworks", *Transportation Science* 23, 166–176.

Dror, M., G. Laporte and P. Trudeau (1993), "Vehicle Routing with Split Deliveries", *Discrete Applied Mathematics*, forthcoming.

Dror, M. and P. Trudeau (1986), "Stochastic Vehicle Routing with Modified Savings Algorithm", *European Journal of Operational Research* 23, 228–235.

Dror, M. and P. Trudeau (1990), "Split Delivery Routing", *Naval Research Logistics* 37, 383–402.

Golden, B.L. and A.A. Assad (1988), *Vehicle Routing: Methods and Studies*, North-Holland, Amsterdam.

Laporte, G. (1992), "The Vehicle Routing Problem: an Overview of Exact and Approximate Algorithms" *European Journal of Operational Research* 59, 345–358.

Laporte, G., M. Desrochers and Y. Nobert (1984), "Two Exact Algorithms for the Distance Constrained Vehicle Routing Problem", *Networks* 14, 161–172.

Laporte, G. and Y. Nobert (1987), "Exact Algorithms for the Vehicle Routing Problem", in *Surveys in Combinatorial Optimization*, pp. 147–184, S. Martello, G. Laporte, M. Minoux and C. Ribeiro (eds.), North-Holland, Amsterdam.

Larson, R.C. (1988), "Transportation of Sludge to the 106 Mile Site: An Inventory Routing Algorithm for Fleet Sizing and Logistic System Design, *Transportation Science* 22, 186–198.